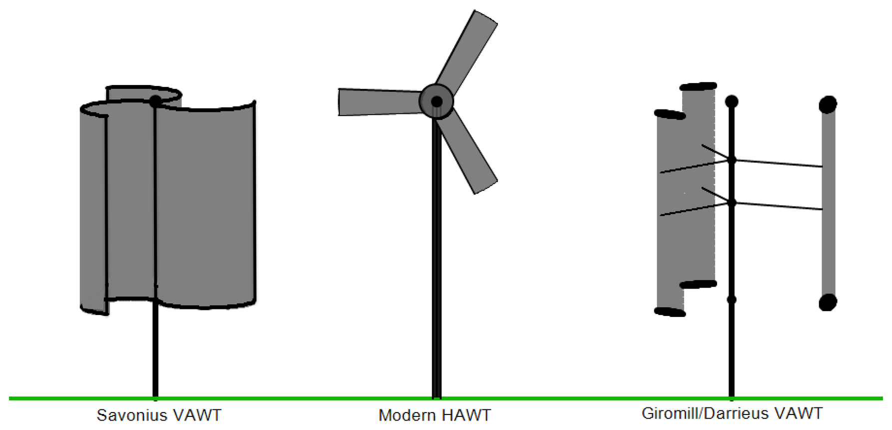
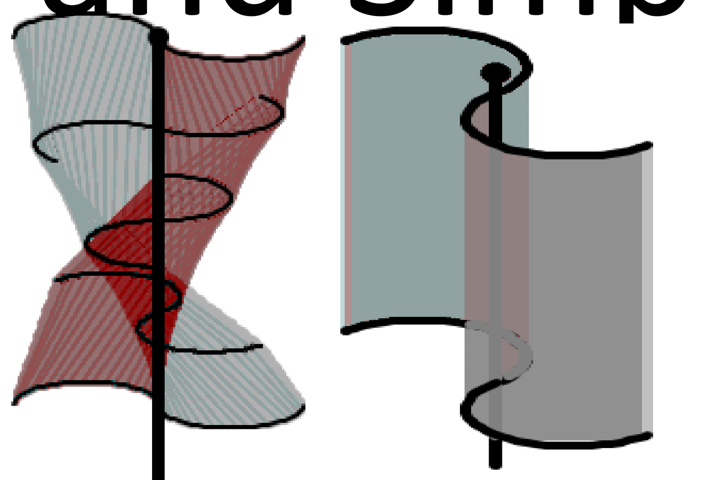
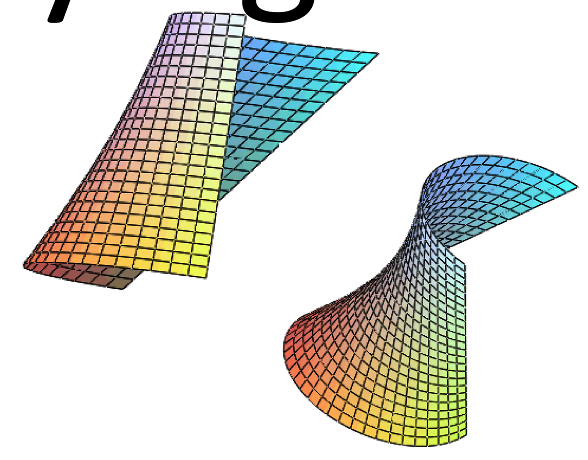
Modeling the Twisted Savonius Wind Turbine Geometrically and Simplifying its Construction



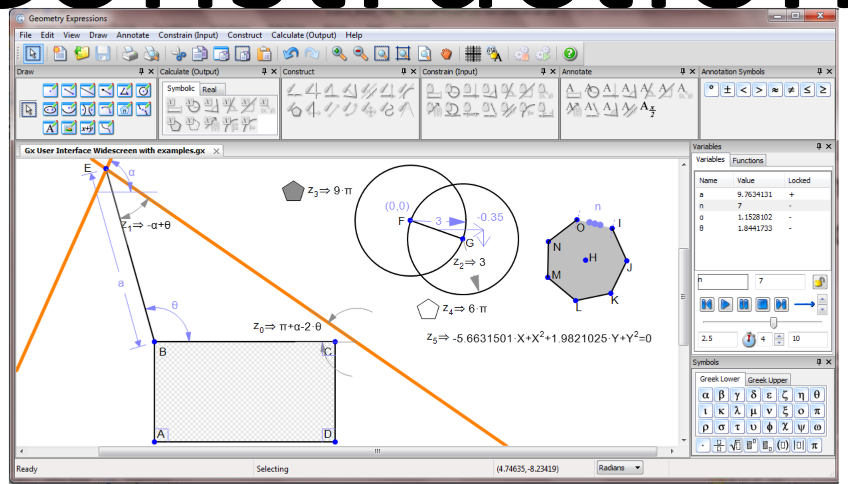
Three Basic Types of Wind Turbine



Twisted and Untwisted Savonius



Blade in *Maple*



Geometry Expressions User Interface

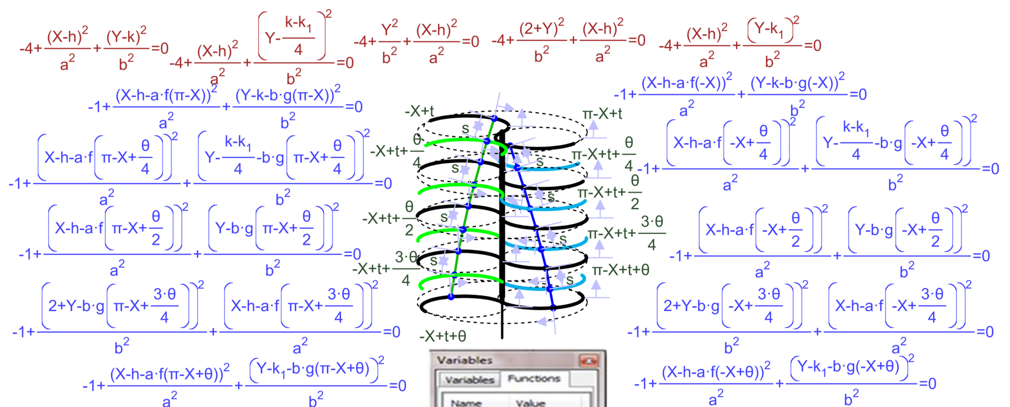
Goals

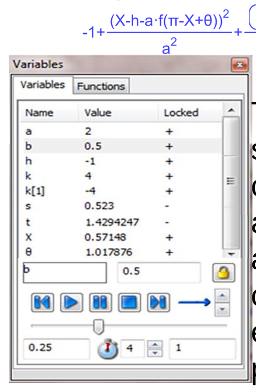
Investigate the shape and simplify the construction method of the Twisted Savonius Wind Turbine

Background

- Currently, most wind turbines are Horizontal Axis (HAWTs) and found in windy areas away from the majority of the people who use their power
- Vertical Axis Wind Turbines (VAWTs) are rarely used in industrial applications because of their relative inefficiency in the open plain [3]
- The most logical place to generate power is as near to the location of use as possible, to reduce transmission losses
- The best place near buildings for wind turbines is on top of them, because higher wind speeds are found at higher altitudes [6] and airflow around buildings directs the air over them [4]
- Wind is very turbulent around buildings because they interrupt airflow [2]
- Therefore, placement in the path of the strongest, turbulent winds and use of a turbine that is operable in turbulent winds are essential
- Traditional HAWTs and most VAWTs cannot operate in turbulent winds [9]
- One VAWT design has shown a lot of potential in turbulent, rooftop settings: the Twisted Savonius [5, 8, 10]
- The Savonius turbine consists of vertical scoops oriented around the axle and operated by the wind pushing on the scoops, making it drag-based [1]
- Twisting the design allows the wind to be pushing on the turbine for more of the rotation, if two blades are twisted 180° or pi radians, then the wind will be utilized for the full rotation of the turbine
- This scenario is ideal for power generation: the Twisted Savonius design on the roof of a power-using building, but…
- The Twisted Savonius blade is extremely difficult to manufacture, requiring advanced metal work or molding
- By exploring the properties of the blade shape, it can be optimized to maximize surface area and potential to capture wind
- Simplification of the shape would help find a new method for building the blade allowing much more widespread use of this design
- Like a sphere, the blades cannot be unrolled into a flat surface
- However, triangles can be used to fold a flat surface into an approximation of the blade shape
- Increasing the # of triangles will give a more and more accurate approximation; as the # of triangles approaches infinity, the approximation approaches the actual blade shape

Symbolic Framework for Full Model



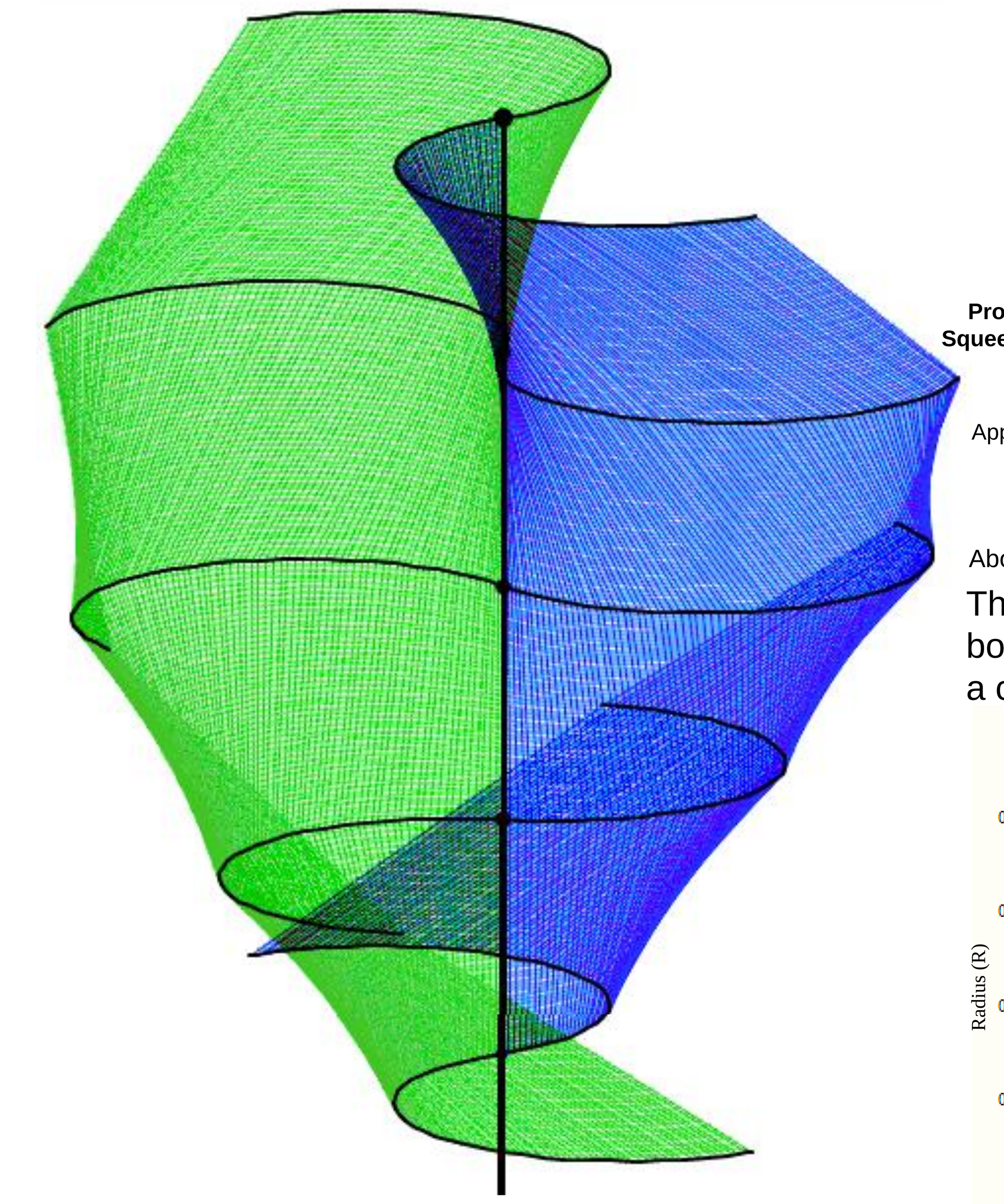


The figure above shows the framework, within the symbolic geometry software *Geometry Expressions*, that comprises the full wind turbine model. All of the variables and functions are defined in the variables tool panel (left, above) where they can be adjusted and animated. Color coding indicates the types of constraints: large stationary ellipse equations are red, moving ellipses are blue, and proportional points (on lines/around ellipses) are green.

Geometry Expressions (Gx)

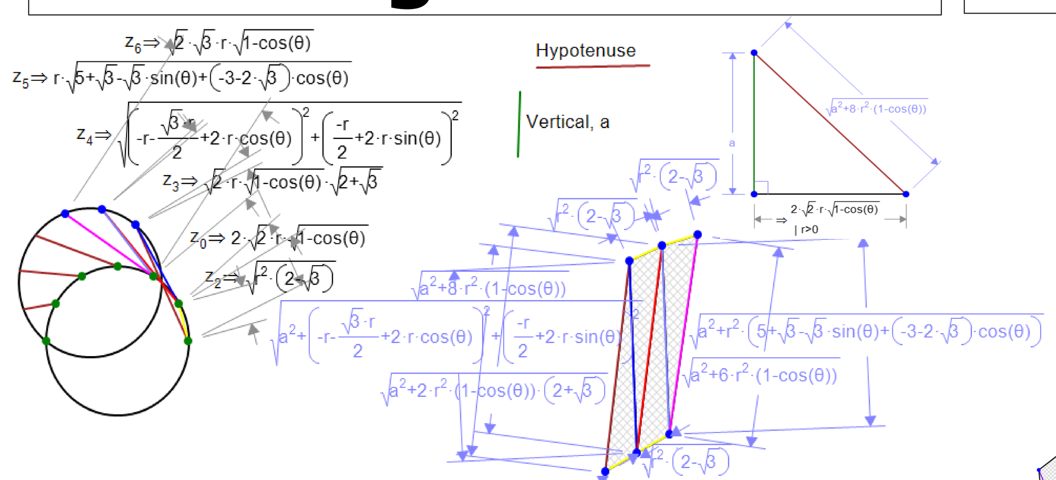
- *Geometry Expressions (Gx)* is a symbolic geometry software program which allows the user to build figures constrained by numbers and variables and make calculations on them
- The biggest advantage is that the radius, height, angle of twist, and other variables can be adjusted and even animated to help observe the shape with various parametric adjustments
- The disadvantage is that the program constructs figures in the two dimensional plane, and the turbine is very three dimensional in shape
- Mathematical tricks can be used to accurately build the turbine in the software and make it seem as if it is in a three-dimensional plane
- The software can be best utilized if it is paired with a computer algebra system (CAS), such as *Maple*, which can manipulate and graph the expressions created in *Gx* [7]

The Turbine in Gx



Using ellipses, proportional points, loci and traces, this model of the Twisted Savonius VAWT was developed. It is fully constrained mathematically, facilitating dimensional adjustments, and can be animated to spin as if in operation or to twist and untwist from the simple Savonius design.

Unrolling the Blades



Because of the geometric planar relations of the blade shape, it cannot be unrolled into a flat surface, like a sphere. Instead, triangles were used to approximate the turbine's shape in the top view model. Incorporating the vertical dimension, hidden in this view, to the side lengths of these triangles, using the Pythagorean Theorem, allowed the turbine shape to be approximated by a flat surface which can be folded into the turbine shape. Using more triangles for this approximation produces a more accurate figure that can be folded up to create what is essentially the actual shape of the blade. With enough triangles, this model should be able to be rolled onto a turbine frame without folding at every triangle, but this step has yet to be completed due to the complexity of the models. Some sample triangle side lengths from the 32 side approximation are shown below, calculated from the top view, then constrained on the unrolled model with the Pythagorean Theorem incorporating the height (a).

Calculations:

$$z_{59} \Rightarrow \sqrt{2}\cdot r\cdot\sqrt{2+2\cdot\cos\left(\frac{29\cdot\pi}{32}\right)-2\cdot\cos(\theta)-\cos\left(\frac{29\cdot\pi}{32}+\theta\right)-\cos\left(\frac{29\cdot\pi}{32}-\theta\right)}$$

$$z_{60} \Rightarrow \sqrt{\left(-r-r\cdot\cos\left(\frac{15\cdot\pi}{16}\right)+r\cdot\cos(\theta)+r\cdot\cos\left(\frac{29\cdot\pi}{32}+\theta\right)\right)^2+\left(-r\cdot\sin\left(\frac{15\cdot\pi}{16}\right)+r\cdot\sin(\theta)+r\cdot\sin\left(\frac{29\cdot\pi}{32}+\theta\right)\right)^2}$$

$$z_{63} \Rightarrow \sqrt{2}\cdot r\cdot\sqrt{2+2\cdot\cos\left(\frac{31\cdot\pi}{32}\right)-2\cdot\cos(\theta)-\cos\left(\frac{31\cdot\pi}{32}+\theta\right)-\cos\left(\frac{31\cdot\pi}{32}-\theta\right)}$$

Constraints:

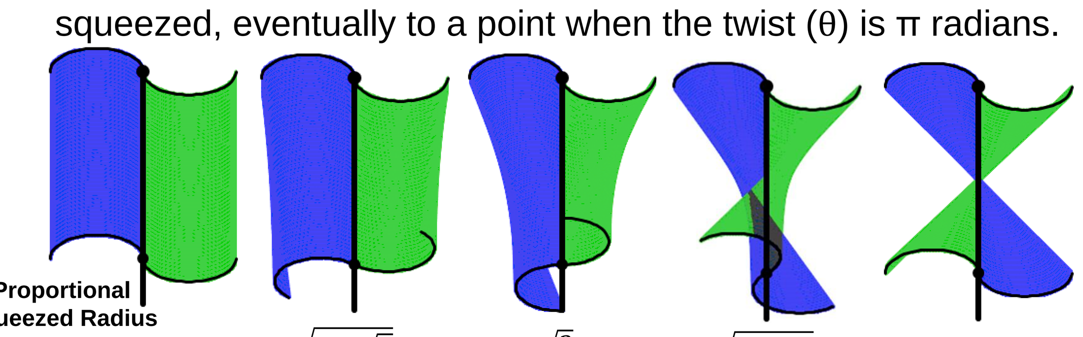
$$\sqrt{a^2+2\cdot r^2\cdot\left(2+2\cdot\cos\left(\frac{29\cdot\pi}{32}\right)-2\cdot\cos(\theta)-\cos\left(\frac{29\cdot\pi}{32}+\theta\right)-\cos\left(\frac{29\cdot\pi}{32}-\theta\right)\right)}$$

$$\sqrt{a^2+\left(-r-r\cdot\cos\left(\frac{15\cdot\pi}{16}\right)+r\cdot\cos(\theta)+r\cdot\cos\left(\frac{29\cdot\pi}{32}+\theta\right)\right)^2+\left(-r\cdot\sin\left(\frac{15\cdot\pi}{16}\right)+r\cdot\sin(\theta)+r\cdot\sin\left(\frac{29\cdot\pi}{32}+\theta\right)\right)^2}$$

$$\sqrt{a^2+2\cdot r^2\cdot\left(2+2\cdot\cos\left(\frac{31\cdot\pi}{32}\right)-2\cdot\cos(\theta)-\cos\left(\frac{31\cdot\pi}{32}+\theta\right)-\cos\left(\frac{31\cdot\pi}{32}-\theta\right)\right)}$$

The Squeeze

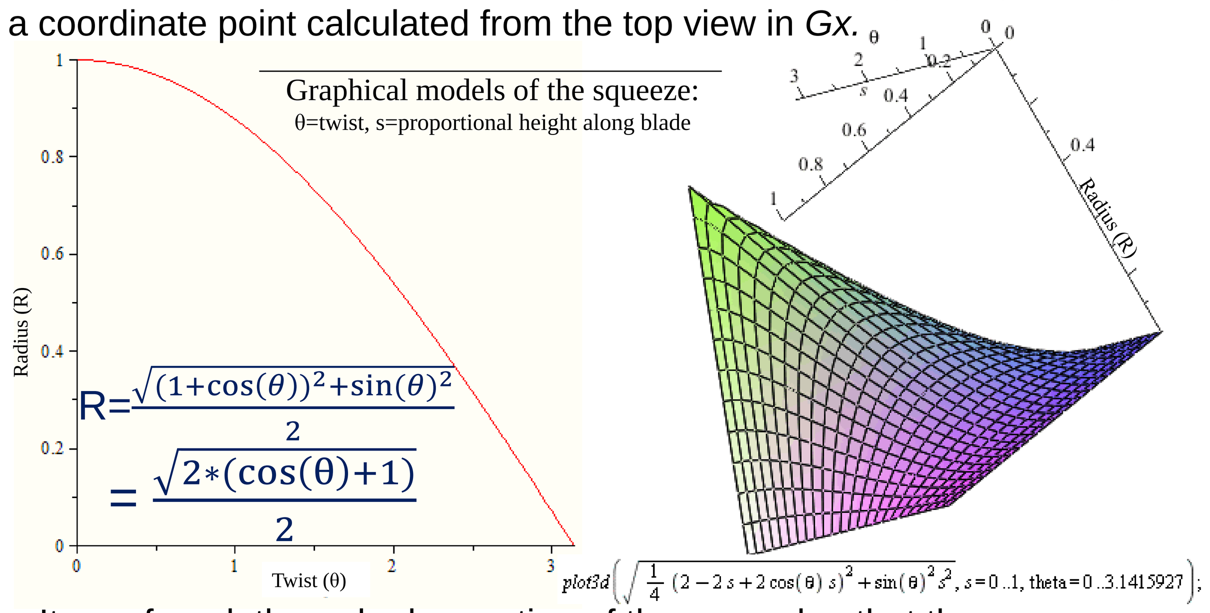
When the Savonius VAWT is twisted, the surface of the blade is squeezed, eventually to a point when the twist (θ) is π radians.



Proportional Squeezed Radius					
Exact:	1	$\frac{\sqrt{2+\sqrt{2}}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2-\sqrt{2}}}{2}$	0
Approximate:	1.	.9238795	.7071068	.38268343	0.
θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π

Above: Savonius wind turbine twisting from 0 to π radians with numerical squeeze measurements.

This squeeze was modeled in the computer algebra system *Maple*, in both two-dimensional and three-dimensional formats, by manipulating a coordinate point calculated from the top view in *Gx*.



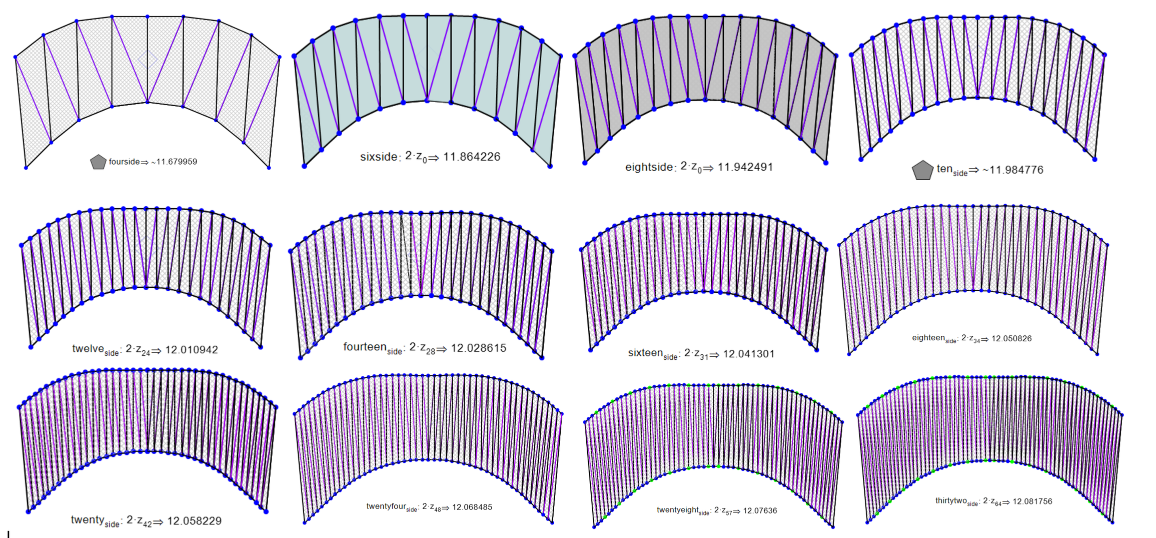
$$R=\frac{\sqrt{(1+\cos(\theta))^2+\sin(\theta)^2}}{2}$$

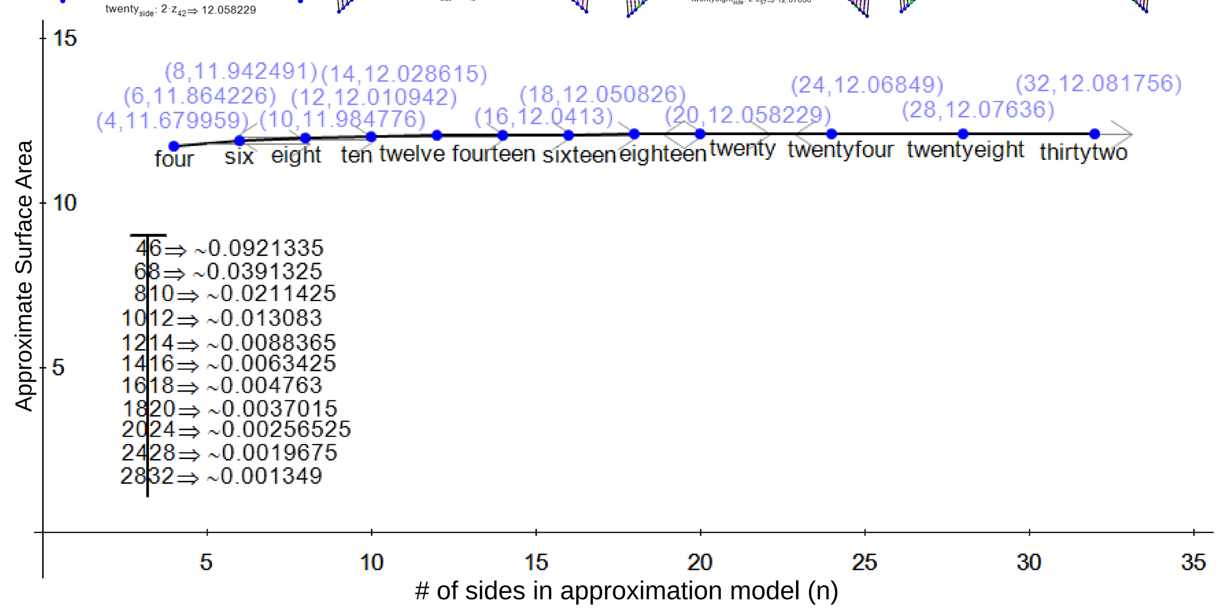
$$=\frac{\sqrt{2*(\cos(\theta)+1)}}{2}$$

It was found, through observation of these graphs, that the squeeze can be most efficiently avoided by stacking four sections of turbine, each twisted pi/4 radians. This minimizes squeeze and complexity of construction while maximizing twist, and therefore functionality.

Finding the Surface Area Limit

Triangle approximation models were created using 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 28, and 32 sides. The symbolic area of these figures could not be calculated by Gx (not even an individual triangle could be), so the surface area in terms of the twist angle theta could not be found. Instead, the approximate values were found by using more and more triangles to approach the limit of the surface area for specific twist angles and height : radius (a:r) ratios. Below is the series of completed approximations, in number-of-sides order, at an a:r ratio of 1.83:1 and a twist of pi/4 radians. A graph of the surface areas follows.





Abstract

The drag-based Twisted Savonius Vertical Axis Wind Turbine (VAWT) has shown promising applications for use on the tops of buildings, enabling clean energy production at the site of its use, virtually eliminating transportation losses. Unfortunately, the turbine's shape is very complex and three-dimensional because of its twist, requiring complex machinery to construct. I was able to model the geometry of the shape with the symbolic geometry program Geometry Expressions, developing visual models that depict the appearance of the turbine in operation and show the effects of twisting the blade. Ellipses, loci and traces comprised the visual model. The most significant finding was that the radius of the turbine is squeezed as the turbine is twisted, which occurs because of the geometric principles of the blade, not just the limitations of the materials. A greater angle of twist results in a greater potential efficiency in operation. Utilizing the calculus principles of definite integrals allowed creation of an approximation of the shape, "unrolled" into a flat shape using triangles. This can be used to build the turbine much more simply and, with refinement, could allow widespread use of Twisted Savonius turbines on rooftops with little cost relative to other alternative energy options.

Discussion

- Discovering and modeling the squeeze has led to knowledge of exactly how to build the geometry of the turbine
- The squeeze has also simplified the construction method, by limiting the sections of true Savonius shape to 4, instead of approaching the limit of infinity
- It also explains limits to manually twisting the turbine, as found in previous studies [5,8] that go beyond the physical limitations of the materials alone
- The visuals created enable proper visualization of the turbines and can be combined with pictures (insert-able in *Gx*) to create renderings of how the turbines would look on buildings—*in operation!*
- Knowing the surface area of the blades allows cost estimates to be made, based on the materials used
- A 2 meter tall blade (for a 3 meter turbine) could be built from the triangle approximations for under $250 (with axle-mount, without gears, generator, electronics) and most of this cost would go toward the turbine's metal frame
- The cheaper cost and simpler construction open the door for widespread use of the Twisted Savoinus Design
- The surface area for each of the 4 pieces that comprise each blade is ~.82 units squared when there is an overall a:d ratio of 2:1 (a:r of 0.25:0.5 per section) and the twist is pi/4 radians for each section of each side
- Therefore, the total surface area for the fully twisted model at the simplest logical a:d ratio is ~3.28 units squared, a useful figure to know for efficiency and material costs purposes

Citations

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All figures created by author unless otherwise noted. User interface of *Geometry Expressions* by Saltire Software, image/figure by author.